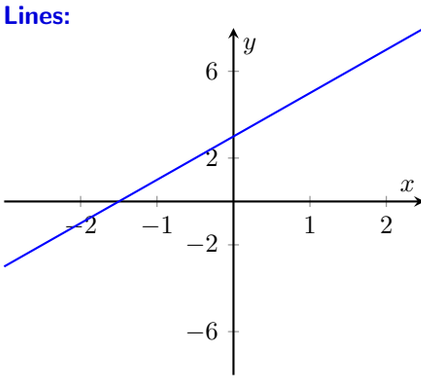
Week 1 – Precalculus I

Coordinate System

Lines:



- Slope: $m = \dfrac{\Delta y}{\Delta x}$
- slope-intercept equation: $y = mx + n$
- point-slope equation: $y - y_1 = m(x - x_1)$

If two lines are parallel, $m_1 = m_2$. If two lines are perpendicular, $m_1 \cdot m_2 = -1$

Exercise 1-1: Find the equations of the following lines:

a) Passing through origin and $(1, 10)$

b) Passing through $(2, 9)$ and $(4, 13)$

c) Passing through $(2, 4)$, parallel to $3x+5y = 1$

d) Passing through $(1, 1)$ and perpendicular to $2y - x - 1 = 0$

Circle: The unit circle is:

$$x^2 + y^2 = 1$$

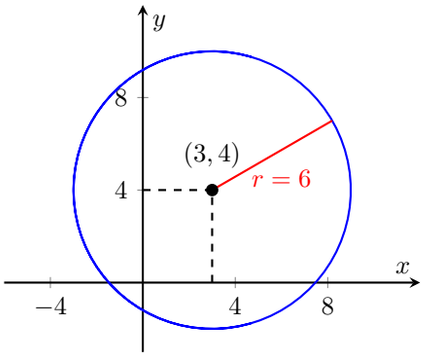
In general, the equation of the circle with center (a, b) and radius R is:

$$(x - a)^2 + (y - b)^2 = R^2$$

For example, the circle

$$(x - 3)^2 + (y - 4)^2 = 6^2$$

looks like this:



Exercise 1-2: Find the equation and sketch the circle with center at $(2, 4)$, passing through origin.

Functions

A function f defined on a set D of real numbers is a rule that assigns to each number x in D exactly one real number, denoted by $f(x)$.

Example: $f = x^4,\ f = e^{-x^2},\ f = |x|$

Domain, Range: The set D of all numbers for which $f(x)$ is defined is called the domain of the function f. The set of all values of $f(x)$ is called the range of f.

Intervals:

- closed interval: $[a, b] = \{x : a \leqslant x \leqslant b\}$
- open interval: $(a, b) = \{x : a < x < b\}$
- half-open interval: $(a, b] = \{x : a < x \leqslant b\}$
- unbounded interval: $(a, \infty) : \{x : a < x\}$

Exercise 1-3: Find the domains of the following functions:

a) $f(x) = \sqrt{x - 16}$

b) $f(x) = \dfrac{1}{\sqrt{x - 16}}$

c) $f(x) = \dfrac{x}{6 - x - x^2}$

Exercise 1-4: Find the domain and range of the following functions:

a) $f(x) = \dfrac{1}{x}$

b) $f(x) = x^2$

c) $f = x^2 - 16$

d) $f = (x - 4)^2$

Graph: The graph of an equation is the set of all points (x, y) that satisfy the equation.

Vertical Line Test: A vertical line does not intersect the graph of a function at more than one point.

Parabolas: The function $f(x) = ax^2 + bx + c$ is called a quadratic function. Its graph is a parabola. We can write the same equation as

$$f = a(x - h)^2 + k$$

to find the vertex and draw the parabola.

Exercise 1-5: Sketch the graph of the following parabolas:

a) $f(x) = x^2 - 2x + 4$

b) $f(x) = x^2 - 16$

c) $f(x) = -x^2 + 2x + 8$

d) $f(x) = 12x - 3x^2$

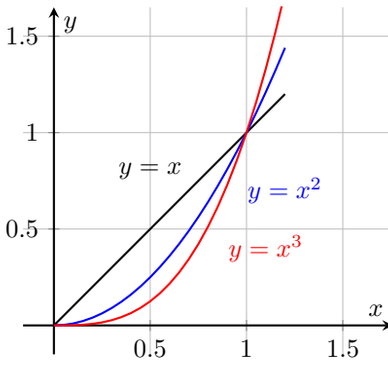
Power Functions: A function of the form $f = x^k$ where k is a constant is called a power function.

Exercise 1-6: Sketch the functions

$$y = x,\ y = x^2,\ y = x^3$$

for $0 < x < 1$ on the same coordinate system.

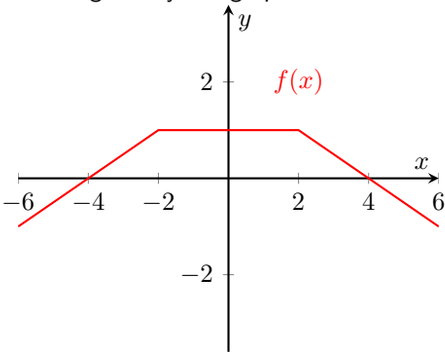
Solution:



Piecewise-Defined Functions: We may define a function using different formulas for different parts of the domain. For example, the absolute value function is:

$$|x| = \begin{cases} x & x \geqslant 0 \\ -x & x < 0 \end{cases}$$

Exercise 1-7: Find the formula of the following function given by the graph:



Exercise 1-8: Sketch the graphs of the following functions:

a) $f(x) = |x|$

b) $f(x) = |x - 4|$

c) $f(x) = |x^2 - 4|$

d) $f(x) = \sqrt{x}$

e) $f(x) = \sqrt{|x|}$

f) $f(x) = \dfrac{1}{x}$

g) $f(x) = \begin{cases} x^2 & x \leqslant 0 \\ x/2 & x > 0 \end{cases}$

h) $f(x) = \begin{cases} 2x + 4 & x < 1 \\ 5 - x & x \geqslant 1 \end{cases}$

Exercise 1-9: Sketch the graphs of the following functions:

a) $f(x) = x^2$ and $g(x) = x^3$

b) $f(x) = x^2 + 8$ and $g(x) = x^3 + 8$

c) $f(x) = x^2 - 8$ and $g(x) = x^3 - 8$

d) $f(x) = (x + 2)^2$ and $g(x) = (x + 2)^3$

e) $f(x) = (x - 2)^2$ and $g(x) = (x - 2)^3$

f) $f(x) = 2x^2$ and $g(x) = 2x^3$

f) $f(x) = x^2/10$ and $g(x) = x^3/10$

Polynomials: A function of the form

$$p(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$$

is called a polynomial of degree n.

Example: $120x^5 - 17x + \dfrac{7}{2}$ is a polynomial.

$\sqrt{x}, x^{-1}, \dfrac{1}{1+x}$ are NOT.

Rational Functions: The quotient of two polynomials is a rational function $f(x) = \dfrac{p(x)}{q(x)}$.

What is the domain of a polynomial? A rational function?

Example: Sketch the graph of the following functions:

a) $f = |x| + x$

b) $f = \begin{cases} x, & x < 2 \\ x^2, & x \geqslant 2 \end{cases}$

c) $f = \dfrac{1}{x}$

d) $f = \dfrac{1}{x^2}$

e) $f = \dfrac{1}{1 - x^2}$

Quadratic Equations: The solution of the equation

$$ax^2 + bx + c = 0$$

is:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \text{where} \qquad \Delta = b^2 - 4ac$$

Here, we assume $a \neq 0$.

If $\Delta > 0$ there are two distinct solutions.
If $\Delta = 0$ there is a single solution.
If $\Delta < 0$ there is no solution.
(We only consider the real solutions)

Exercise 1-10: Solve $3x^2 + 6x + 1 = 0$

Review Exercises

Exercise 1-11: Are the domains of the functions f and $1/f$ the same?

Exercise 1-12: Find the domain and range of the following functions:

a) $f(x) = -1 - \sqrt{(5 - x^2)}$

b) $f(x) = |-5 + x^2|$

c) $f(x) = x(1 - x)$

d) $f(x) = \dfrac{1}{1 + x^2}$

Exercise 1-13: Find the equation of the line passing through origin and making an angle of 150° with the positive $x-$axis.

Exercise 1-14: Find the intersection point of the lines $3x + y = 1$ and $y = 2x + 16$. Sketch the lines.

Exercise 1-15: Find the solution of the following equations:

a) $x^2 - 7x + 6 = 0$

b) $2x^2 + 5x - 3 = 0$

c) $12x^2 - 7x + 1 = 0$

d) $x^2 - 4x + 1 = 0$

Exercise 1-16: Let $f(x) = x(x - 8)$.

a) Sketch the graph of f. Show intersection points with coordinate axes.

b) Sketch the graph of $f + 4$

c) Sketch the graph of $-f$

d) Sketch the graph of $2f$

e) Sketch the graph of $|f|$

— End of WEEK —

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