1) Find the derivative of the following functions. Explain your solution.

a. $f(x) = \dfrac{5^x}{3} + \left[1 + \cos^7(\arctan^2(x))\right]^8.$

$$f'(x) = \frac{5^x \ln 5}{3} + 8\left[1+\cos^7(\arctan^2 x)\right]^7$$
$$\cdot(7\cos^6(\arctan^2 x))$$
$$\cdot(-\sin(\arctan^2 x))$$
$$\cdot(2\arctan x)$$
$$\left(\frac{1}{1+x^2}\right)$$

b. Find $\dfrac{dy}{dx}$ at $(0, \pi)$ if $x^2\cos^2 y - \sin y = 0$.

$$2x\cos^2 y - x^2 . 2\cos y \sin y . y' - \cos y . y' = 0$$

At $(0, \pi)$ we have;

$$\underbrace{2.0\cos^2\pi}_{=0} - \underbrace{0^2 . 2\cos\pi \sin\pi y'}_{=0} - \cos\pi . y' = 0$$

$$-\cos\pi . y' = 0 \quad \Rightarrow \quad y' = 0.$$

c. $f(x) = (\sin x)^{\sec x}$.

$$y = (\sin x)^{\sec x}$$

$$\ln y = \ln(\sin x)^{\sec x} = \sec x \ln(\sin x)$$

Take der of both sides;

$$\frac{y'}{y} = \left(\sec x \tan x . \ln \sin x + \sec x . \frac{\cos x}{\sin x}\right)$$

$$y' = (\sin x)^{\sec x}\left(\sec x \tan x . \ln \sin x + \sec x . \cot x\right)$$

2) Evaluate the following limits. Explain your solution.

a. $\lim_{x \to 0+} \sin x \cdot \ln x.$ $\left(\frac{\infty}{\infty}, LH\right)$

$$\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}} \overset{\uparrow}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{(\sin x)^2}}$$

$$= \lim_{x \to 0^+} - \underbrace{\frac{\sin x}{x}}_{1} \cdot \underbrace{\frac{\overbrace{\sin x}^{=0}}{\cos x}}_{1}$$

$$= 0$$

b. $\lim_{x \to \infty} \left(\frac{x+2}{x-1}\right)^x.$

we have 1^∞ indeterminate form.

$$\lim_{x \to \infty} \ln\left(\frac{x+2}{x-1}\right)^x = \ln L$$

$$\lim_{x \to \infty} x \ln\left(\frac{x+2}{x-1}\right) = \lim_{x \to \infty} \frac{\ln\left(\frac{x+2}{x-1}\right)}{\frac{1}{x}}$$

$$\overset{\Downarrow}{=} \lim_{x \to \infty} \frac{\frac{x-1}{x+2} \cdot \frac{-3}{(x-1)^2}}{-\frac{1}{x^2}}$$

$\left(\frac{0}{0}, LH\right)$

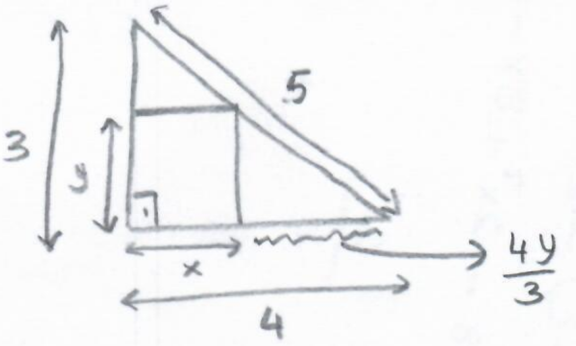
$$= \lim_{x \to \infty} \frac{3x^2}{(x+2)(x-1)}$$

$$= 3$$

$$\ln L = 3 \Rightarrow L = e^3$$

3) This question has two unrelated parts.

a. Find the dimensions of the rectangle, x and y, with the largest area that can be inscribed in the right triangle given by the figure.



Find maximum of $x.y$.

$x + \frac{4y}{3} = 4 \Rightarrow x = 4 - \frac{4y}{3}$.

$A = \left(4 - \frac{4y}{3}\right) \cdot y$

$= 4y - \frac{4y^2}{3}$

$A' = 4 - \frac{8y}{3} = 0 \Rightarrow y = \frac{3}{2}$

	3/2	
A'	+ ↗	− ↘

For $\boxed{y = \frac{3}{2}}$ A has a maximum.

$x + \frac{4y}{3} = 4 \Rightarrow \boxed{x = 2}$

b. Find an equation of the tangent line to the curve $y = x + \sin x$ at $(0, 0)$.

Find y' at $(0,0)$ to find m.

$y' = 1 + \cos x$

at $(0,0)$ $y' = 1 + 1 = 2$ (slope)

$y - y_0 = m(x - x_0)$

$y - 0 = 2(x - 0)$

$y = 2x$

4) Sketch the graph of $f(x) = \dfrac{x^2-3}{x-2}$. Show all your work in detail, i.e., find the domain of the function, find asymptotes (if exist), find critical and inflection points and indicate local maximum and local minimum.

(1) Domain $= \mathbb{R} - \{2\}$

(2) Asymptotes ; • oblique

$$\begin{array}{r|l} x^2-3 & x-2 \\ \hline -x^2+2x & x+2 \\ 2x-3 & \\ -2x+4 & \\ \hline 1 & \end{array}$$

$f(x) = x+2+\dfrac{1}{x-2}$

$\boxed{y = x+2}$ is obl. asy.

• vertical $\lim_{x\to 2^+} f(x) = \infty$ $\lim_{x\to 2^-} f(x) = -\infty$

$\boxed{x=2}$ is v.a.

• No hor. as.

(3) $f'(x) = \dfrac{2x(x-2)-(x^2-3)}{(x-2)^2} = \dfrac{x^2-4x+3}{(x-2)^2} = \dfrac{(x-3)(x-1)}{(x-2)^2}$

$x=3$
$x=1$
$x=2$ (double r.)

$f''(x) = \dfrac{(2x-4)(x-2)^2-(x^2-4x+3)2.(x-2)}{(x-2)^4} = \dfrac{2}{(x-2)^3}$

$x=2$ (3 roots)

$f(1) = 2 \to$ local max

$f(3) = 6 \to$ local min.

	1		2	3	
f'	+ ↗	− ↘	− ↘		+ ↗
f''	− ∩		+ ∪		
	(	)	(		)