1. Evaluate the following integrals

1a. $\int \sec^2 x \tan^2 x dx$
Solution
$u = \tan x \Rightarrow du = \sec^2 x dx$
$\Rightarrow \int \sec^2 x \tan^2 x dx = \int u^2 du = \frac{u^3}{3} + c = \frac{1}{3}\tan^3 x + c.$
1b. $\int_0^{\pi/2} (\sin 2x)^2 \cos x dx.$
Solution
$I = \int (\sin 2x)^2 \cos x dx = \int (2\sin x \cos x)^2 d(\sin x) dx$
$u = \sin x$
$I = 4\int u^2 (1 - u^2) du = \frac{4}{3}u^3 - \frac{4}{5}u^5$
$= \frac{4}{3}\sin^3 x - \frac{4}{5}\sin^5 x + c$
$\int_0^{\pi/2} (\sin 2x)^2 \cos x dx$
$= \frac{4}{3}\sin^3 x - \frac{4}{5}\sin^5 x \Big|_0^{\pi/2}$
$= \frac{4}{3} - \frac{4}{5} = \frac{8}{15}.$

2. Evaluate the derivative of $f(x) = (g(x))^{g(x)}$, where

$$g(x) = \int_0^{x^2} e^{-t^2} dt.$$

Solution
$g'(x) = \frac{d}{dx}\int_0^{x^2} e^{-t^2} dt = e^{-x^4} 2x,$
$\ln f = g \ln g \Rightarrow \frac{f'}{f} = g' \ln g + g\frac{g'}{g} = g'(\ln g + 1) \Rightarrow$
$f' = fg'(\ln g + 1)$

$$= 2e^{-x^4} x \left(\int_0^{x^2} e^{-t^2} dt\right)^{\int_0^{x^2} e^{-t^2} dt} \left(\ln \int_0^{x^2} e^{-t^2} dt + 1\right).$$

3. Evaluate the following integrals
3a. $\int (9 - x^2)^{1/2} dx.$
Solution
$x = 3\sin\theta \Rightarrow dx = 3\cos\theta d\theta,$
$\theta = \arcsin\frac{x}{3},$
$\int (9 - x^2)^{1/2} dx = 9\int (1 - \sin^2\theta)^{1/2} \cos\theta d\theta$
$= 9\int \cos^2\theta d\theta = \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta + c$
$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{9}{4}\sin\left(2\arcsin\frac{x}{3}\right) + c$
$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{9}{4}\frac{2}{3}x\sqrt{1 - \frac{1}{9}x^2}$
$= \frac{9}{2}\arcsin\frac{1}{3}x + \frac{1}{2}x\sqrt{9 - x^2}.$
Check
$\frac{d}{dx}\left(\frac{9}{2}\arcsin\frac{x}{3} + \frac{9}{4}\frac{2}{3}x\sqrt{1 - \frac{1}{9}x^2}\right) = \sqrt{9 - x^2}.$

3b. $I = \int_0^1 (x+1)^{1/2} \ln(x+1)\, dx$
Solution
$u = \ln(x+1)$, $dv = (x+1)^{1/2}\, dx$,
$du = \frac{dx}{x+1}$, $v = \int (x+1)^{1/2}\, dx = \frac{2}{3}(x+1)^{\frac{3}{2}}$,
$\int (x+1)^{1/2} \ln(x+1)\, dx = \frac{2}{3}(\ln(x+1))(x+1)^{\frac{3}{2}} - \int \frac{2}{3}\sqrt{x+1}\, dx$
$= \frac{2}{3}(\ln(x+1))(x+1)^{\frac{3}{2}} - \frac{4}{9}(x+1)^{\frac{3}{2}}$.
$I = \frac{2}{3}(\ln(1+1))(1+1)^{\frac{3}{2}} - \frac{4}{9}(1+1)^{\frac{3}{2}} + \frac{4}{9}$
$= \frac{4}{3}\sqrt{2}\ln 2 - \frac{8}{9}\sqrt{2} + \frac{4}{9}$.
4. Find the integral $\int \frac{dx}{(x+2)(x^2+2)}$.
Solution
$\frac{1}{(x+2)(x^2+2)} = \frac{1}{6(x+2)} - \frac{\frac{1}{6}x - \frac{1}{3}}{x^2+2}$
$\int \frac{dx}{6(x+2)} = \frac{1}{6}\ln(x+2)$
$\int \frac{\frac{1}{6}x - \frac{1}{3}}{x^2+2}\, dx = \frac{1}{12}\ln\left(x^2+2\right) - \frac{1}{6}\sqrt{2}\arctan\frac{1}{2}\sqrt{2}x$.

5. Sketch the finite region enclosed by the curves $y = x^{-1}$, $y = -x+3$ and find its area.
Solution $x^{-1} = -x+3$, $x_1 = \frac{3}{2} - \frac{1}{2}\sqrt{5}$, $x_2 = \frac{1}{2}\sqrt{5} + \frac{3}{2} \Rightarrow$
$A = \int_{\frac{3}{2}-\frac{1}{2}\sqrt{5}}^{\frac{1}{2}\sqrt{5}+\frac{3}{2}} \left(-x+3-x^{-1}\right) dx = \ln\left(\frac{3}{2} - \frac{1}{2}\sqrt{5}\right) - \ln\left(\frac{1}{2}\sqrt{5} + \frac{3}{2}\right) + \frac{3}{2}\sqrt{5}$.