1) Evaluate the following integrals.

a) $\displaystyle\int \frac{5x^4 - 7x^{3/5} + 2}{x^3}\,dx.$

$$= \int \left(5x - 7x^{-12/5} + 2x^{-3}\right) dx$$

$$= 5\frac{x^2}{2} - 7\frac{x^{-7/5}}{-7/5} + 2\frac{x^{-2}}{-2} + C$$

b) $\displaystyle\int (3x + 4)(\sqrt{x} + 1)\,dx.$

$$= \int \left(3x^{3/2} + 3x + 4x^{1/2} + 4\right) dx$$

$$= 3\frac{x^{5/2}}{5/2} + 3\frac{x^2}{2} + 4\frac{x^{3/2}}{3/2} + 4x + C$$

c) $\displaystyle\int (\tan^2 x + 1)\,dx.$

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \sec^2 x\,dx = \tan x + C$$

2) a) Evaluate the following integral.

$$\int (x^3 - x)e^{x^2-1}\,dx.$$

$a = x^2 - 1$

$da = 2x\,dx$

$$= \int \frac{1}{2} a\, e^{a}\, da$$

$u = a \qquad dv = e^{a} da$

$du = da \qquad v = e^{a}$

$$= \frac{1}{2}\left[a e^{a} - \int e^{a} da\right]$$

$$= \frac{1}{2}\left(a e^{a} - e^{a}\right) + C$$

$$= \frac{1}{2}\left((x^2-1)e^{x^2-1} - e^{x^2-1}\right) + C$$

b) Evaluate the following limit.

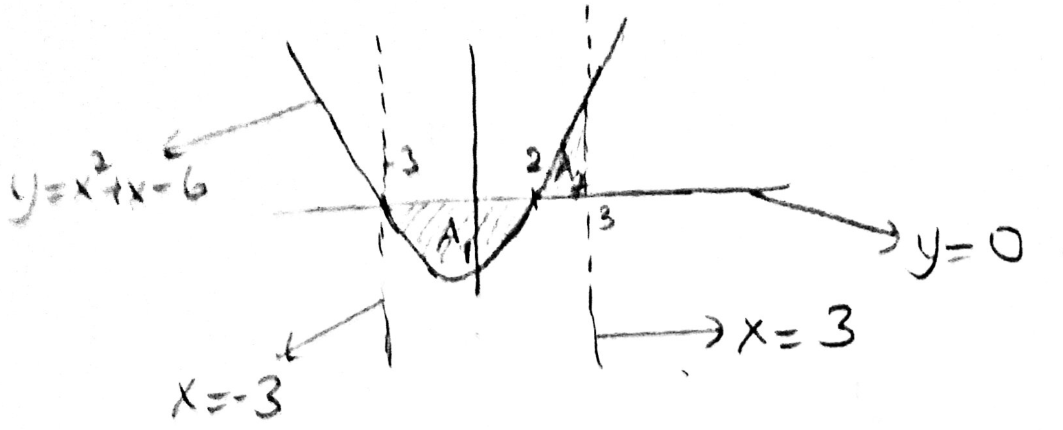
$$\lim_{x\to 1} \frac{\displaystyle\int_{x^3}^{x} e^{t^2}\,dt}{x^2 - 1}$$

Apply L'Hopital's rule

$$\lim_{x\to 1} \frac{\frac{d}{dx}\left(\int_{x^3}^{x} e^{t^2} dt\right)}{2x}$$

$$= \lim_{x\to 1} \frac{e^{x^2}.1 - e^{x^6}.3x^2}{2x} = \frac{-2e}{2} = -e$$

3) Find the area between $y = x^2 + x - 6$, $x = -3$, $x = 3$ and $y = 0$.



Area $= A_1 + A_2$

$$A_1 = \int_{-3}^{2} \left(0 - (x^2+x-6)\right)dx = \frac{125}{6}$$

$$A_2 = \int_{2}^{3} \left((x^2+x-6) - 0\right)dx = \frac{17}{6}$$

Area $= \dfrac{142}{6}$

4) Evaluate the following integrals.

a) $\int \cos^3 x \sin^5 x\,dx$.

$u = \sin x$

$du = \cos x\,dx$

$\cos^2 x = 1 - \sin^2 x$

$= 1 - u^2$

$= \int (1-u^2)\, u^5\, du$

$= \int (u^5 - u^7)\, du$

$= \frac{u^6}{6} - \frac{u^8}{8} + C$

$= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$

b) $\int \sec^3 x \tan^3 x\,dx$.

$u = \sec x$

$du = \sec x \tan x\,dx$

$\tan^2 x = \sec^2 x - 1$

$= u^2 - 1$

$= \int u^2 (u^2 - 1)\, du$

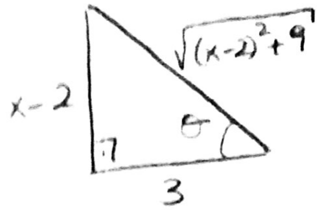
$= \int (u^4 - u^2)\, du$

$= \frac{u^5}{5} - \frac{u^3}{3} + C$

$= \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$

5) Evaluate the following integrals.

a) $\int \frac{x+2}{x^2-4x+13}dx$



$\tan\theta = (x-2)/3$

$\sec^2\theta d\theta = \frac{dx}{3}$

$\theta = \arctan\left(\frac{x-2}{3}\right)$

$$= \int \frac{x+2}{(x-2)^2+9} dx$$

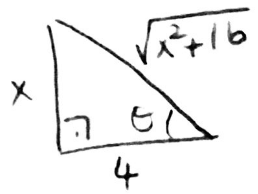
$$= \int \frac{(3\tan\theta + 4)\, 3\sec^2\theta d\theta}{9\sec^2\theta}$$

$$= \frac{1}{3}\int (3\tan\theta + 4)\, d\theta$$

$$= \ln(\sec\theta) + \frac{4}{3}\theta$$

$$= \ln\frac{\sqrt{(x-2)^2+9}}{3} + \frac{4}{3}\arctan\left(\frac{x-2}{3}\right)$$

b) $\int \frac{1}{x^2\sqrt{16+x^2}}dx$



$\tan\theta = \frac{x}{4}$

$\sec^2\theta d\theta = \frac{dx}{4}$

$$= \int \frac{4\sec^2\theta d\theta}{16\tan^2\theta\, 4\sec\theta}$$

$$= \frac{1}{16}\int \frac{\sec\theta}{\tan^2\theta} d\theta$$

$$= \frac{1}{16}\int \frac{\cos\theta}{\sin^2\theta} d\theta$$

$u = \sin\theta$

$du = \cos\theta d\theta$

$$= \frac{1}{16}\int \frac{1}{u^2} du$$

$$= -\frac{1}{16u} + C$$

$$= -\frac{1}{16\sin\theta} + C = -\frac{\sqrt{x^2+16}}{16x} + C$$

6) Evaluate the following integrals.

a) $\int \frac{8x-13}{(x-1)^2(x^2+4)}dx.$ $= \int \left(\frac{2}{x-1} + \frac{-1}{(x-1)^2} - \frac{2x+1}{x^2+4}\right) dx = I$

$$\frac{8x-13}{(x-1)^2(x^2+4)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4}$$

$(x-1)(x^2+4)$ (x^2+4) $(x-1)^2$

$8x-13 = A(x-1)(x^2+4) + B(x^2+4) + (Cx+D)(x-1)^2$

$x=1$ $\quad -5 = 5B \qquad B = -1$

$x=0$ $\quad -13 = -4A - 4 + D \quad \Rightarrow \quad -9 = D - 4A \quad \Rightarrow \quad 4A - 9 = D.$

$x=2$ $\quad 3 = 8A - 8 + 2C + D \Rightarrow 11 = 8A + 2C + D \Rightarrow 20 = 12A + 2C$

$x=-1$ $\quad -21 = -10A - 5 - 4C + 4D \Rightarrow -16 = -10A - 4C + 4D \Rightarrow 20 = 6A - 4C$

$A = 2$

$C = -2$

$D = -1$

$$I = 2\ln|x-1| + \frac{1}{(x-1)} - \ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C.$$

b) $\int_0^\infty te^{-st}dt, \quad s > 0.$

$= \lim_{R\to\infty} \int_0^R te^{-st}\,dt$ $\qquad u = t \quad dv = e^{-st}dt$

$du = dt \quad v = \frac{e^{-st}}{-s}$

$$= \lim_{R\to\infty} \left[t\frac{e^{-st}}{-s}\Big|_0^R + \int_0^R \frac{e^{-st}}{s}dt \right]$$

$$= \lim_{R\to\infty} \left[\frac{Re^{-Rs}}{-s} + \frac{e^{-st}}{-s^2}\Big|_0^R \right]$$

$$= \lim_{R\to\infty} \left[\frac{Re^{-Rs}}{-s} + \frac{e^{-Rs}}{-s^2} - \frac{1}{-s^2} \right] = \frac{1}{s^2}$$