1) Find the domain of the following functions.

a) $f(x) = (\ln(1 - \ln(1 - x^2)))^{1/2}$.

$\sqrt{\ln(1-\ln(1-x^2))} \geq 0 \Rightarrow \ln(1-\ln(1-x^2)) \geq 0$

$\Rightarrow 1 - \ln(1-x^2) \geq 1$

$\ln(1-x^2) \leq 0$

$\Rightarrow 0 < 1-x^2 \leq 1$

$\Rightarrow -1 < -x^2 \leq 0$

$0 \leq x^2 < 1 \Rightarrow -1 < x < 1$

b) $f(x) = \dfrac{1}{(x-1)^2 + 2}$.

Denominator should not be equal to zero.

$(x-1)^2 + 2 \neq 0$ ... $(*)$

Since $(x-1)^2 \geq 0$, the condition given by $(*)$ is never satisfied.

Domain $= \mathbb{R}$.

c) $f(x) = -(x-5)^6 + 2$.

The function is defined everywhere

Domain $= \mathbb{R}$.

2) Find the line which is perpendicular to the line

$$\frac{x}{2} + \frac{y}{3} = 1$$

and the circle $(x-4)^2 + (y-5)^2 = 1$.

Firstly, find the slope of the line

$$3x + 2y = 6$$

$$y = \frac{6-3x}{2} \Rightarrow m_1 = -\frac{3}{2}$$

The the line we are seaching for is perpendicular, its slope, m_2, and multiplication of m_1 is equal to -1

$$m_1 \cdot m_2 = -1 \Rightarrow m_2 = \frac{2}{3}$$

To find line equation we need a point on this line.

It passes through the center of the circle.

Center of the circle is $(4,5)$.

$$y - y_0 = m(x - x_0)$$

$$y - 5 = \frac{2}{3}(x-4)$$

$$y = \frac{2x}{3} + \frac{7}{3}.$$

3) Evaluate the following limits if they exist. Explain your solution. Do not use L'Hôpital's rule.

a) $\lim_{x\to\infty}\left((3x^2+1)^{1/2}-(3x^2-2x-1)^{1/2}\right)$.

$$\lim_{x\to\infty}\left(\sqrt{3x^2+1}-\sqrt{3x^2-2x-1}\right)\cdot\frac{\sqrt{3x^2+1}+\sqrt{3x^2-2x-1}}{\sqrt{3x^2+1}+\sqrt{3x^2-2x-1}}$$

$$=\lim_{x\to\infty}\frac{2x+1}{x\sqrt{3+\frac{1}{x^2}}+x\sqrt{3-\frac{2}{x}-\frac{1}{x^2}}}$$

$$=\lim_{x\to\infty}\frac{2x+1}{2\sqrt{3}x}=\frac{1}{\sqrt{3}}$$

b) $\lim_{x\to 0}\sin(x^2)\cot(3x^2)$.

$$\lim_{x\to 0}\frac{\sin x^2}{x^2}\,x^2\cos 3x^2\cdot\frac{3x^2}{\sin 3x^2}\cdot\frac{1}{3x^2}$$

$$=\frac{1}{3}$$

c) $\lim_{x\to 5^-}\frac{|2x-10|(x+2)}{x-5}$.

Since $x\to 5^-$ we have $|2x-10|=10-2x$

$$\lim_{x\to 5^-}\frac{(10-2x)(x+2)}{x-5}=\lim_{x\to 5^-}-2(x+2)=-14$$

4) If exist, find all horizontal, vertical and oblique asymptotes for the following functions. Explain your solution.

a) $f(x) = \dfrac{x^3 + x^2 + 2}{x^2 - x - 1}$

There is no horizontal asymptote.

$$\begin{array}{l|l} x^3 + x^2 + 2 & x^2 - x - 1 \\ \hline -x^3 + x^2 + x & x + 2 \\ 2x^2 + x + 2 & \\ -2x^2 + 2x + 2 & \\ \hline 3x + 4 & \end{array}$$

$f(x) = x + 2 + \dfrac{3x+4}{x^2-x-1}$

we have oblique asymptote $y = x + 2$.

Check where the func. is undefined. $(x^2 - x - 1 = 0)$

$x_{1,2} = \dfrac{1 \pm \sqrt{1+4}}{2}$

$\lim\limits_{x \to \frac{1+\sqrt{5}}{2}} \dfrac{x^3 + x^2 + 2}{x^2 - x - 1} = \infty$ $\quad \lim\limits_{x \to \frac{1-\sqrt{5}}{2}} \dfrac{x^3 + x^2 + 2}{x^2 - x - 1} = \infty$

we have two vertical asymptotes $x = \dfrac{1 \pm \sqrt{5}}{2}$.

b) $f(x) = \dfrac{|3x + 1|}{27x^3 + 1}$

$\lim\limits_{x \to \infty} \dfrac{|3x+1|}{27x^3+1} = \lim\limits_{x \to \infty} \dfrac{3x+1}{27x^3+1} = 0 = \lim\limits_{x \to -\infty} \dfrac{-3x-1}{27x^3+1}$

Horizontal asymptote $y = 0$.

There is no oblique asymptote.

For $x = -1/3$ $f(x)$ is undefined.

$\lim\limits_{x \to -1/3^+} \dfrac{3x+1}{27x^3+1} = \lim\limits_{x \to -1/3^+} \dfrac{1}{9x^2+3x+1} = 1.$

$\lim\limits_{x \to -1/3^-} \dfrac{-3x-1}{27x^3+1} = \lim\limits_{x \to -1/3^-} \dfrac{-1}{9x^2+3x+1} = -1.$

There is no vertical asymptote.